Code No: R21043 **R10** SET - 1

II B. Tech I Semester, Regular Examinations, Nov – 2012
PROBABILITY THEORY AND STOCHASTICS PROCESSES
(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
~~~~~~~~~~~~~~~~~~~~~~~~~~~~

1. a) Distinguish between Joint probability and conditional probability.
   b) State and Prove the Baye's Theory.
   c) A box of 30 diodes is known to contain 5 defective ones. If two diodes are slaved at random without replacement, what is the probability that at least one of these diodes is defective?

2. a) Define the probability density function of random variable and state its properties
   b) Explain the Rayleigh distribution and density functions.

3. a) A random variable has pdf

$$f_y(x) = \frac{5}{4}(1 - x^4) \qquad 0 \le x \le 1$$
$$= 0 \qquad \text{else where}$$

   Find E (4x+2) and E(x^2)
   b) Find the characteristic function of the random variable x having the density function.

$$f_x(x) = \frac{1}{2a} \quad |x| < a$$
$$= 0 \quad \text{other wise}$$

4. a) Determine the constant b such that the function

$$f_{XY}(x, y) = 3xy \qquad 0 < x < 1, \quad 0 < y < b$$
$$= 0 \qquad \text{other wise}$$

   is a valid joint density function
   b) Find the density of w=x+y where the densities of x and y are assumed to be

$$f_X(x) = [u(x) - u(x-1)]$$
$$f_Y(y) = [u(y) - u(y-1)]$$
~~~~~~~~~~~~~~~~~~~~~~~~~~~~

5. a) Random variable X and Y have the joint density.

$$f_{xy}(x, y) = \frac{1}{24} \qquad 0 < x < 6, \quad 0 < y < 4$$
$$= 0 \qquad else\ where$$

What is the expected value of the functions g (x,y) = $(xy)^2$

b) Random variable x_1 and x_2 have the joint Characteristic function

$$\phi\, x_1, x_2(w_1, w_2) = \left[(1 - j2w_1)(1 - j2w_2)\right]^{-N/2}$$

Where N>0 is an integer

Find the correlation and moment's m_{20} and m_{02}

6. a) Explain the following: i) concept of Stationarity ii) Ergodic Random processes.

b) State and prove the properties of Auto correlation function.

7. a) If the Auto Correlation of random binary transmission process is given by

$$R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T} & |\tau| \le T \\ 0 & \text{else where} \end{cases}$$

Find the power spectral density of process

b) Write short notes on "power density spectrum".

8. a) A random noise x(t) having the power spectrum $S_{XX}(w) = \frac{3}{49 + w^2}$ is applied to a network for which h(t)=u(t) t^2 exp(-7t). The network response is denoted by y(t)

i) What is the average power in x(t)

ii) Find the power spectrum of x (t)

iii) Find the average power of y (t)

b) Write short notes on "Arbitrary Noise Sources".

Code No: R21043 **R10** SET - 2

II B. Tech I Semester, Regular Examinations, Nov – 2012
PROBABILITY THEORY AND STOCHASTICS PROCESSES
(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
~~~~~~~~~~~~~~~~~~~~~~~~~~~~

1. a) Explain the following in brief
   i) Probability as a relative frequency ii) Joint provability
   iii) Baye's theorem iv) Independent events

   b) A multi channel microwave link is to provide te1ephone communication to a remote community having 12 subscribes, each of whom uses the link 20% of time during peak hours. How many channels are needed to make the line available during peak hours to
   i) Eighty percent of the subscribes all of the time
   ii) All of the subscribes 80% of the time
   iii) All of the subscribes 95% of the time

2. a) Find the value of the constant K so that

$$f_x(x) = \begin{cases} Kx^2(1-x^3) & 0 \le x \le 1 \\ 0 & \text{other wise} \end{cases}$$

   Is this a proper density function of a continuous random variable.

   b) Explain about the Gaussian distribution and density function.

3. a) Find the moment generating function about the origin of the Poisson distribution.

   b) Show that any characteristic function q_x (w) satisfies

$$|q_x(w)| \le q_x(0) = 1$$

4. a) Let X and Y be jointly continuous random variables with joint pdfs

$$fXY(x,y) = x^2 + \frac{xy}{3} \quad for\ 0 \le x \le 1, \quad 0 \le y \le z$$
$$= 0 \qquad \text{other wise}$$

   i) Are x and y independent
   ii) Check f(x/y) and f(y/x) are pdfs or not

   b) State and prove the centered limit theorem.
~~~~~~~~~~~~~~~~~~~~~~~~~~~~

5. a) A joint density is given as

$$f_{XY}(x, y) = \begin{cases} x(y+1.5) & 0 < x < 1 \text{ and} \\ 0 & \text{other wise} \end{cases}$$

Find all the joint moments m_{nk}. n and k=0,1,………

b) Let x be random variable with E(x)=2 and var (x)=3 verify that random variable x and the random variable y=-4x+8 are orthogonal.

6. a) Consider a random process x(t)=A cos ($\omega t+\theta$) where w and θ are constants and A is a random variable with zero mean and variance σ_A^2. Determine whether x(t) is a wide sense stationary process or not

b) Distinguish between Auto correlation function and cross correlation function. State the properties of cross correlation function.

7. a) A stationary random process has an auto correlation function of

$R_x(\tau) = 16 - e^{-5|\tau|} \cos 20\pi\tau + 8 \cos 10\pi\tau.$

i) Find the variance of this process

ii) Find the spectral density of this process

b) Prove that the power spectrum and Autocorrelation function of the random process form a Fourier Transform pair.

8. Write short notes as: i) Thermal Noise ii) Properties of band limited processes

Code No: R21043 **R10** SET - 3

II B. Tech I Semester, Regular Examinations, Nov – 2012
PROBABILITY THEORY AND STOCHASTICS PROCESSES
(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
~~~~~~~~~~~~~~~~~~~~~~~~~~~

1. a) Explain the following
   i) Axioms of probability ii) Independent events iii) Total probability

   b) A and B are events such that $p(A \cup B) = \frac{3}{4}$ $p(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{2}{3}$ find $p(\overline{A}/B)$

   c) In a group of external number of men and women 10% men and 45% women are unemployed. What is the probability that a person selected at random is employed?

2. a) If the communicative distribution function of a random variable x is given by

$$f_X(x) = \begin{cases} 1 - \frac{4}{x^2} & x \geq 2 \\ 0 & x \leq 2 \end{cases}$$

   Find i) $p(x<3)$ ii) $p(4<x<5)$ iii) $p(x\geq 3)$

   b) Explain about Poisson distribution and density functions.

3. a) Let x be a random variable with distribution f_x given by

$$f_X(x) = \begin{cases} 1 - e^{-\lambda x} & 0 \leq x \leq \infty \\ 0 & \text{other wise} \end{cases}$$

   Find the pdf of x. Determine the mean and variance of the distribution.

   b) Find the moment generating function (MGF) for the distribution $f_x(x)=\frac{1}{2^x}$ x=1,2,3……..

   Also find its mean.
~~~~~~~~~~~~~~~~~~~~~~~~~~~

4. a) The pdf is given by

$$f_{XY}(x,y) = \frac{6}{5}(x+y^2) \quad 0 \le x \le 1, \quad 0 \le y \le 1$$
$$= 0 \qquad \text{other wise}$$

i) Find the marginal pdf of x and that of y

ii) Find $p\left[\frac{1}{4} \le y \le \frac{3}{4}\right]$

b) Explain about conditional distribution and density internal conditioning.

5. a) Let(x,y) be two dimensional random variable described by joint pdf.

$$f_{XY}(x,y) = 8xy \quad 0 \le x \le 1, \quad 0 \le y \le x$$
$$= 0 \qquad \text{else where}$$

Find the cov(x,y)

b) Explain about "jointly Gaussian random variables"

6. a) Discus the stationarity of the random process $x(t) = A\cos(\omega_0 t + \theta)$ where A and ω_0 are constants and θ is a uniformity distributed random variable in $(0,2\pi)$

b) Consider two random processes x(t)=3 cos (wt+θ) and y(t)=2 cos(wt+θ-$\pi/2$) where θ a random variable distributed in $(0,2\pi)$

prove that $\left|R_{xy}(\tau)\right| \le \sqrt{R_{xx}(0)\, R_{yy}(0)}$

7. a) The psd of a random process is given by

$$S_{xx}(w) = \begin{cases} \pi & |w| < 1 \\ 0 & \text{else where} \end{cases}$$

Find its Auto correlation function

b) What is cross power density spectrum? State its properties

8. a) Find the output power density spectrum and output Auto correlation function for a system with

$$h(t) = e^{-t} \quad t \ge 0 \text{ for as input with psd} = \frac{ho}{2} \qquad -\infty < t < \infty$$

b) Write short notes on "effective noise temperature".

Code No: R21043 **R10** SET - 4

II B. Tech I Semester, Regular Examinations, Nov – 2012
PROBABILITY THEORY AND STOCHASTICS PROCESSES
(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
~~~~~~~~~~~~~~~~~~~~~~~~~~

1. a) Define the following and give are example for each of the following
   i) Discrete and continuous sample space
   ii) Mutually Exclusive event
   iii) Equally likely event
   b) A letter is known to have come either from LONDON or CLIFTON. On the post card only two consecutive letters as 'ON' are legible what is the chance that it came from LONDON.
   c) What is the probability that a randomly selected integer chosen from the first 100 positive integers is odd?

2. a) The distribution for x is defined by $f_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 - \frac{1}{4} e^{-x} & \text{for } x \geq 0 \end{cases}$

   Determine p(x=0) and p(x>0)

   b) Explain about "Binominal distribution and density functions"

3. a) A continuous distribution is given by

$$fx(x) = \frac{1}{x\sqrt{2\pi}} e^{-(\log x)^2} \quad x \geq 0$$
$$= 0 \qquad elsewhere$$

   Find the mean, standard deviation, coefficient of skewness of this distribution

   b) The characteristic function for a Gaussian random variable x, having R mean value of zero is $q_X(w) = \exp\left|-j\sigma^2 w^2/2\right|)$. Find all the moments of x using q_X (w)
~~~~~~~~~~~~~~~~~~~~~~~~~~

4. a) The joint pdt of the two dimensional r.v(x,y) is given by

$$fxy(x,y) = \frac{1}{4} e^{|x| - |y|} \quad -\infty < x < \infty, \; -\infty < y < \infty$$
$$= 0 \quad \text{else where}$$

i) Check whether x and y are independent

ii) $p(x \le 1, y \le 0)$

b) The probability density function of two statistically independent random variables X and Y are

$$fx(x) = \begin{cases} \frac{3}{32}(4 - x^2) & -2 \le x \le 2 \\ = 0 & \text{else where in } x \end{cases}$$

$$fy(y) = \frac{1}{2}[x(y+1) - u(y-1)]$$

Find the exact probability density of the sum w=x+y

5. a) Two random variable have a uniform density on a circular region defined by

$$fxy(x,y) = \frac{1}{\pi r^2} \quad x^2 + y^2 \le r^2$$
$$= 0 \quad \text{else where}$$

Find the mean value of the function $g(x,y)=x^2+y^2$

b) Two random variables x and y have the joint characteristic function

$$q_{xy}(w_1, w_2) = \exp\left(-2w_1^2 - 8w_2^2\right)$$

Show that i) x and y are both zero mean random variable ii) x and y are uncorrelated

6. a) Explain the following w.r.to Random processes

i) Strict sense stationary ii) Mean Ergodic processes.

b) Explain about Poisson Random processes

7. a) For a random process x(t), assume that $Rxx(\tau) = \rho\, e^{-\tau^2/2a^2}$ where $\rho>0$ and $a>0$ are constants. Find the power density spectrum of x(t)

b) Prove that the cross power spectrum and cross correlation function of Random process form a Fourier transform pair.

8. a) Define a given system and derive a relation between the input and output spectra of the system.

b) Explain "Resistive noise" in detail.