Code: R7210402 **R07**

B.Tech II Year I Semester (R07) Supplementary Examinations, May 2013

PROBABILITY THEORY & STOCHASTIC PROCESSES

(Common to ECE & ECC)

Time: 3 hours Max. Marks: 80

Answer any FIVE questions
All questions carry equal marks

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1 (a) State and prove Baye's theorem of probability.

(b) A_1, A_2, A_3 are three mutually exclusive and exhaustive sets of events associated with a random experiment E_1. Events B_1, B_2, B_3 are mutually exclusive and exhaustive sets of events associated with random experiment E_2. The joint probabilities of occurrence of these events are listed in table given below.

	B_1	B_2	B_3
A_1	$^3/_{36}$	$*$	$^5/_{36}$
A_2	$^5/_{36}$	$^4/_{36}$	$^5/_{36}$
A_3	$*$	$^6/_{36}$	$*$
$P(B_j)$	$^{12}/_{36}$	$^{14}/_{36}$	$*$

(i) Find the missing probabilities ($*$) in the table.

(ii) Find P (B_3/A_1) and P (A_1/B_3).

(iii) Are events A_1 and B_1 statistically independent?

2 (a) Consider the probability density $f_X(x) = ae^{-b|x|}$ where x is a random variable whose values range from $x = -\infty \ to \ \infty$. Find

(i) The CDF $F_X(x)$. (ii) The relationship between a and b.

(iii) The probability that the outcome x lies between 1 and 2.

(b) The probability of a defective pen from a pen manufacturing unit is 3%. In a box of 400 pens, find the probability that;

(i) Exactly 3 pens are defective. (ii) More than 2 pens are defective.

3 (a) State and prove properties of moment generating function of a random variable X.

(b) A random variable X has a characteristic function given by $\emptyset_X(\omega) = \begin{cases} 1-|\omega|, & |\omega| \le 1 \\ 0 & , |\omega| > 1 \end{cases}$

Find density function.

4 (a) The joint pdf of two random variables X and Y is $f_{X,Y}(x,y)$= K(6- x- y) for o< x <2, 2<y<4

= 0 else where

Find (i) The value of K. (ii) P (X + Y < 3).

(b) If $f_{X,Y}(x,y) = xy\exp\left(-\left(\frac{x^2+y^2}{2}\right)\right) for\ x > 0, y > 0$

= 0 other wise

(i) Are X and Y statistically independent. (ii) Find $P(X \le 1, Y \le 1)$.

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5 (a) Show that the variance of weighted sum of uncorrelated random variables equals the weighted sum of the variances of the random variables.

(b) Two Gaussian random variables X_1 and X_2 have zero means and variances ${\sigma_{X_1}}^2 = 4$ and ${\sigma_{X_2}}^2 = 9$. Their covariance $C_{X_1X_2} = 3$. If X_1 and X_2 are linearly transformed to new variables Y_1 and Y_2 according to $Y_1 = X_1 - 2X_2$ and $Y_2 = 3X_1 + 4X_2$. Find means, variances and covariance of Y_1 and Y_2.

6 (a) A random process is defined as $X(t) = A\sin(\omega t + \theta)$ where A is a constant and θ is a random variable uniformly distributed over $(-\pi, \pi)$. Check X(t) for stationarity.

(b) If a random process $X(t) = \sin(\omega t + y)$ where Y is a random variable uniformly distributed in the interval $(0, 2\pi)$. Prove that $cov\ (t_1, t_2) =$ auto correction $(t_1, t_2) = \frac{cos\omega(t_1 - t_2)}{2}$.

7 (a) Prove the following:

(i) $R_e[s_{XY}(\omega)]$ is an even function of ω and $I_m[s_{XY}(\omega)]$ is an odd function of ω.

(ii) If X(t) and Y(t) are uncorrelated and have constant means $E[X]$ and $E[Y]$ prove that $s_{XY}(\omega) = s_{YX}(\omega) = 2\pi\ E[X]\ E[Y]\ \delta(\omega)$.

(b) Find the auto correlation function of a process whose PSD is:

$$s_{XX}(\omega) = \frac{157 + 12\omega^2}{(16 + \omega^2)(9 + \omega^2)}.$$

8 (a) What are the precautions to be taken in cascading stages of a network from the point of view of noise reduction?

(b) What is the need for band limiting the signal towards the direction of increasing SVR?

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