

Code: 9ABS401/9ABS304

1

B.Tech II Year II Semester (R09) Regular & Supplementary Examinations, April/May 2013

PROBABILITY & STATISTICS

(Common to CE, ME, CSS and IT)

Time: 3 hours

Max Marks: 70

Answer any FIVE questions
All questions carry equal marks

- 1 Two dice are thrown. Find the probability that the sum of the numbers coming upon them is 9. If it is known that the number 5 always occurs on the first die.
- 2 A random variable x has the density function $F(x) = \begin{cases} 1/4, & -2 < x < 2 \\ 0, & \text{else where} \end{cases}$
- 3 Two dice are thrown X assign to each point if S the sum of the variables on the faces. Find the mean and variance of the random variable.
- 4 (a) The mean of certain normal population is equal to the standard error of the mean of the samples of 64 from that distribution. Find the probability that the mean of the sample size 36 will be negative?
(b) The probability that the sample mean will not differ from the true mean by more than 15% of the standard deviation is 0.95. How large the sample should be?
- 5 (a) Prove that for a random sample of size n , x_1, x_2, \dots, x_n taken from a finite population $S^2 = 1/n \sum_{i=1}^n (x_i - \bar{x})^2$ is not unbiased estimator of the parameter σ^2 but $1/n-1 \sum_{i=1}^n (x_i - \bar{x})^2$ is unbiased.
(b) Assuming that $\sigma = 20$, how large a random sample be taken to assert with probability 0.95 that the sample mean will not differ from the true mean by more than 3.0 points.

Contd. in Page 2

Code: 9ABS401/9ABS304

1

- 6 (a) In a random sample of 400 persons from a large population, 120 are females. Can it be said that males and females are in the ratio 5:3 in the population? Use 1% level of significance.
- (b) An investigation of the merits of two kinds of flash light batteries showed that a random sample of 100 batteries of brand A tested on average 36.5 hrs with a S.D. of 1.8 hrs. While a random sample of 80 batteries of brand B tested on the average 36.8 hrs with a S.D. of 1.5 hrs. Test whether the observed difference between the average life times is significant or not? Use 0.05 level of significance.
- 7 1000 students at a college level were graded according to their I.Q and the economic conditions of their home. Use χ^2 test to find out whether there is any association between condition at home with $\alpha = 0.05$ and I.Q.

Economic condition	High	Low	Total
Rich	460	140	600
Poor	240	160	400
Total	700	300	1000

- 8 Cars arrive at a petrol pump with exponential inter arrival time having mean $1/2$ min. The attendant takes an average of $1/5$ min per car to supply petrol, the service time being exponentially distributed. Determine:
- (i) The average number of cars waiting to be served
 - (ii) The average number of cars in the system and
 - (iii) The proportion of time for which the pump attendant is idle.

Code: 9ABS401/9ABS304

2

B.Tech II Year II Semester (R09) Regular & Supplementary Examinations, April/May 2013

PROBABILITY & STATISTICS

(Common to CE, ME, CSS and IT)

Time: 3 hours

Max Marks: 70

Answer any FIVE questions
All questions carry equal marks

- 1 (a) If A and B are independent events. Then prove A^c and B^c are also independent events.
(b) If A and B are independent events. Then prove A and B^c are also independent.
- 2 If X is a continuous random variable and $Y = ax+b$ prove that $E(1/y) = aE(x) + b$ and $V(y) = a^2 V(x)$.
- 3 Find the mean and the variance of the uniform probability distribution given by:
$$f(x) = \frac{1}{n} \text{ for } x = 1, 2, \dots, n$$
- 4 (a) Find the maximum difference that we can expect with probability 0.95 between the means of sizes 10 and 12 from a normal population if their standard deviations are found to be 2 and 3 respectively.
(b) If two independent random samples of sizes $n_1 = 9$ and $n_2 = 16$ are taken from a normal population. What is the probability that the variance of the first sample will be at least 4 times as large as the variance of the second sample?
- 5 (a) Write a short note on interval estimation and Bayesian estimation.
(b) Measurements of the weights of a random sample of 200 ball bearing made by a certain machine during one week showed a mean of 0.824 and a S.D of 0.042. Find maximum error at 95% confidence interval. Find the confidence limits for the mean if $x = 32$.

Contd. in Page 2

Code: 9ABS401/9ABS304

2

- 6 (a) In a sample of 600 students of a certain college 400 are found to use ball pens. In another college from a sample of 900 students 450 were found to use ball pens. Test whether 2 colleges are significantly different with respect to the habit of using ball pens.
- (b) The mean consumption of food grains among 400 sampled middle class consumers is 380 gms per day per person with a S.D. of 120 gms. A similar sample survey of 600 working class consumers gave a mean of 410 gms with a S.D. of 80 gms. Are we justified in saying that the two classes consume the same quality of food grains? Use 5% level of significance
- 7 (a) In a random sample of 1000 persons from town A, 400 are found to be consumers of wheat. In a sample of 800 from town B are found to be consumers of wheat. Do these data reveal a significant difference between town A and town B, so far as the proportion of wheat consumers is concerned?
- (b) 5 measurements of the output of the following results. (in kgs of materials per one hour of operation) Assuming that both samples have obtained from normal populations, test at 0.01 level of significance if two populations have the same variance.

Unit A	14.1	10.1	14.7	13.7	14.0
Unit B	14.0	14.5	13.7	12.7	14.1

- 8 Consider a single server queuing system with Poisson input and exponential service time. Suppose the mean arrival rate is 3 calling units per hrs with the expected service time as 0.25 hrs and the maximum permissible number of calling units in the system is two. Obtain the steady state probability of the number of calling units in the system and then calculate the expected number in the system.

Code: 9ABS401/9ABS304

3

B.Tech II Year II Semester (R09) Regular & Supplementary Examinations, April/May 2013

PROBABILITY & STATISTICS

(Common to CE, ME, CSS and IT)

Time: 3 hours

Max Marks: 70

Answer any FIVE questions
All questions carry equal marks

- 1 (a) If $A \subseteq B$ then prove that (i) $P(A^c \cap B) = P(B) - P(A)$ (ii) $P(A) \leq P(B)$
(b) If A and B are mutually exclusive events, then prove that $P(A) \leq P(B^c)$.
- 2 A random variable x has the density function: $f(x) = K \cdot \frac{1}{1+x^2}$, if $-\infty < x < \infty = 0$, otherwise determine K and the distributive function.
- 3 In a distribution exactly normal 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution?
- 4 A random sample of size is taken from a normal population with $\mu = 51.4$ and $\sigma = 6.8$. What is the probability that the mean of the sample will
 - (a) Exceed 52.9
 - (b) Fall between 50.5 and 52.3
 - (c) Be less than 50.6.
- 5 (a) Give the difference between the interval estimation and the Bayesian estimation.
(b) The mean weight loss of $n = 16$ grinding balls after a certain length of time in mill slurry is 3.42 grams with a S.D 0.68 grams. Find the maximum error of estimate at 99% confidence interval. Also construct a 99% confidence interval for the true mean weight loss of such grinding balls under the stated conditions.

Contd. in Page 2

Code: 9ABS401/9ABS304

3

- 6 (a) Experience had shown that 20% of a manufactured product is of top quality. In one day's product of 400 articles only 50 are of top quality. Test the hypothesis at 0.05 levels?
- (b) A sample of 900 members has a mean of 3.4 cm and S.D. 2.61 cm is the sample from a large population of mean 3.25 cm and S.D. 2.61 cm. If the population is normal and its mean is unknown find the 95% fiducial limits of true mean?
- 7 (a) A sample poll of 300 voters from district A and 200 voters from district B showed that 56% and 48% respectively, were in favour of a given candidate. At a 0.05 level of significance, test the hypothesis that there is a difference in the districts?
- (b) Two independent samples of 8 and 7 items respectively had the following values of the variables.

Sample I	9	11	13	11	16	10	12	14
Sample II	11	13	11	14	10	8	10	-

Do the estimates of the population variance differ significantly?

- 8 A bank plans to open a single server drive-in banking facility at a certain center. It is estimated that 20 customers will arrive each hour on average. If on average, it requires 2 min to process a customer's transaction, determine
- (i) The proportion of time that the system will be idle;
- (ii) On the average, how long a customer will have to wait before reaching the server;
- (iii) The fraction of customers will have to wait?

Code: 9ABS401/9ABS304

4

B.Tech II Year II Semester (R09) Regular & Supplementary Examinations, April/May 2013

PROBABILITY & STATISTICS

(Common to CE, ME, CSS and IT)

Time: 3 hours

Max Marks: 70

Answer any FIVE questions
All questions carry equal marks

- 1 An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or 8?
- 2 The cumulative distribution function for a continuous random variable x
$$F(x) = \begin{cases} 1 - e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$
 Find the density function $f(x)$.
- 3 If mean = 70, standard deviation is 16. Find
 - (a) $P(38 \leq x \leq 46)$
 - (b) $(82 \leq x \leq 94)$
 - (c) $(62 \leq x \leq 86)$
- 4 When we take a sample from an infinite population what happen to the standard error of the mean if the sample size is (i) Increased from 50 to 200 (ii) Decreased from 225 to 25.
- 5 The mean mark in mathematics in common entrance test will vary from year to year. If this variation of the mean mark is expressed subjectively by a normal distribution with mean $\mu_0 = 72$ and variance $\sigma_0^2 = 5.76$.
 - (i) What probability can we assign to the actual mean being somewhere between 71.8 and 73.4 for the next year's test?
 - (ii) Construct a 95% Bayesian interval for μ if the test is conducted for a random sample of 100 students from the next incoming class yielding a mean mark of 70 with S.D. of 8.
 - (iii) What posterior probability should be assigned to the event of part (i)?

Contd. in Page 2

Code: 9ABS401/9ABS304

4

- 6 (a) A company claims that its light bulbs are superior to those of its main competitor. If a study showed that a sample of $n_1 = 40$ of its bulbs that a mean life time of 647 hrs of continuous use with a S.D. of 27 hrs; while a sample of $n_2 = 40$ bulbs made by its main competitor had a mean life time of 638 hrs of continuous use with a S.D. of 31 hrs. Does this substantiate the claim at 0.05 level of significance?
- (b) In a study designed to investigate whether certain detonators used with explosives in coal mining meet the requirement that at least 90% will ignite the explosive when charged, it is found that 174 of 200 detonators function property. Test the null hypothesis $p = 0.9$ against the alternative hypothesis $p > 0.9$ at 0.05 level of significance.

- 7 Scores obtained in a shooting competition by 10 soldiers before and after intensive training are given below:

Before	67	24	57	55	63	54	56	68	33	43
After	70	38	58	58	56	67	68	75	42	38

Test whether the intensive training is useful at 0.05 level of significance.

- 8 (a) What is the probability distribution of time spent in the $(M/M/1) : (\infty/\text{FIFO})$ Queuing system?
- (b) What is the probability distribution density function of the waiting time distribution for $(M/M/1) : (\infty/\text{FIFO})$ Queuing system?
