II B.Tech II Semester Examinations,APRIL 2011 NUMERICAL METHODS
Common to Mechanical Engineering, Mechatronics, Production Engineering Time: 3 hours

## Answer any FIVE Questions

All Questions carry equal marks

1. The probability integral $P=\sqrt{\frac{2}{\pi}} \int_{0}^{x} \exp \left(-\frac{1}{2} t^{2}\right) d t$ has the following values

| x | 1.0 | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 0.682689 | 0.706828 | 0.728668 | 0.789856 | 0.769861 | 0.788700 |
| Calculate the values of P for $\mathrm{x}=1.235$ |  |  |  |  |  |  |

2. Evaluate the following taking the interval as 1 using finite difference method.
(a) $\Delta e^{x}$
(b) $\Delta \tan ^{-1} x$
(c) $\Delta 3 x$
(d) $2 x / x$ !
3. Find the a curve to the following data

| x | 0 | 2 | 4 |
| :---: | :---: | :---: | :---: |
| y | 5.1 | 10 | 31.1 |

4. Explain the procedure of împroving the accuracy for an ill conditioned system given below.
$a_{11} x_{1}+a_{12} x_{2}+a_{13} x_{3}=b_{1}$
$a_{21} x_{1}+a_{22} x_{2}+a_{23} x_{3}=b_{2}$
$a_{31} x_{1}+a_{32} x_{2}+a_{33} x_{3}=b_{3}$
5. (a) State the Taylor's series formula to find $y\left(x_{1}\right)$ for solving $d y / d x=f(x, y)$ with the initial value of $y(x-0)=y_{0}$.
(b) Solve $d y / d x=\log _{10}(x+y)$ with the initial value of $y(0)=2$ using Euler's method and find the values of $\mathrm{y}(0.2)$ and $\mathrm{y}(0.4)$ using modified Euler's method.

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[8+7]
$$

6. Solve the Poisson's equation $\partial^{2} u / \partial x^{2}+\partial^{2} u / \partial y^{2}=-x^{2} y^{2}$ in the square region bounded by the lines $\mathrm{x}=0, \mathrm{y}=3$ given that $\mathrm{u}=10$ thorough out the boundaries taking $\mathrm{h}=1$.
7. Determine $\mathrm{a}, \mathrm{b}$ and c such that the formula $\int \mathrm{f}(\mathrm{x}) \mathrm{dx}=\mathrm{h}\{\mathrm{af}(0)+\mathrm{bf}(\mathrm{h} / 3)+\mathrm{c}$ $\mathrm{f}(\mathrm{h})\}$ with the limits $\mathrm{x}=0$ to $\mathrm{x}=\mathrm{h}$ is exact for polynomials of as high order as possible and determine the order of the truncation error.
8. (a) State the merits and demerits of Newton-Raphson method.
(b) By Newton-Raphson method find a real root of the following equation $f(\mathrm{x})=\mathrm{x} \sin \mathrm{x}-1$ correct up to four decimal places starting from $\mathrm{x}_{0}=1$


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1. Derive equations to fit an exponential curve of the form $y=b x^{a}$ where $b>0$; by using the method of least squares.
2. For the following system of equations
$10 x+4 y-2 z=12$
$x-10 y-z=-10$
$5 x+2 y-10 z=-3$, show that Jacobi iteration scheme converges. Obtain the Jacobi iteration scheme in matrix form.
3. If $\mathrm{x}(\mathrm{t})$ is analytic inside the close contour C and if $t_{1}, t_{1}, t_{2} \ldots \ldots \ldots . t_{n}$ lie inside C , show that the remainder term in the error formula for polynomial interpolation can be written as:
$\frac{\pi(t)}{2 \pi i} \int_{C} \frac{x(\tau)}{(\tau-t) \pi(\tau)} d \tau$
4. Solve the following boundary value problem with the step length 0.5 and extrapolate $y^{\prime \prime}+4 y+3=0$ with $y(2)=y(-1)=0$.
5. (a) Obtain the solution in the form general formula for Euler's method for solving the differential equation.
(b) Given $d y / d x=x^{2} /\left(y^{2}+1\right)$ with $y(0)=0$, use Picard's method to obtain the y for different values of $\mathrm{x}=0.25$ and 0.5 .
6. By Newton-Raphson method find a real root of the following equation $f(x)=x^{3}$ -$x^{2}+x-2$ correct up to four decimal places starting from $x_{0}=1$.
7. Write the finite difference scheme to solve $u_{x x}=a u_{t}$ with $\mathrm{u}(0, \mathrm{t})=\mathrm{T}_{0}, \mathrm{u}(1, \mathrm{t})=\mathrm{T}_{1}$ and the initial condition as $\mathrm{u}(\mathrm{x}, 0)=\mathrm{f}(\mathrm{x})$ and explain the procedure to solve it.
8. (a) Derive the solution for the Simpson's $3 / 8^{\text {th }}$ rule from the Newton-Cotes formula for solving the integral equation.
(b) A rod is rotating in a plane and the following table gives the angle $\theta$ (radians) through which the rod has turned for various values of time t (seconds).

| $t$ in seconds | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\theta$ in radians | 0 | 0.12 | 0.49 | 1.12 | 2.02 | 3.20 |

Calculate angular velocity and the angular acceleration of the rod when $\mathrm{t}=$ 0.6 s .
$[7+8]$

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1. The error function $\operatorname{erf}(x)$ is defined by the integral $\operatorname{er} f(x)=\frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^{2}} d t$. Approximate $\operatorname{erf}(0.08)$ by linear interpolation in the given table of correctly rounded values. Estimate the total error.

| $x$ | 0.05 | 0.1 | 0.15 | 0.2 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 0.05637 | 0.11246 | 0.16800 | 0.22270 |

2. (a) What is the Error in modified Euler's method? And compare these errors in comparison with the Euler's method.
(b) Solve $y^{\prime}=x y, y(1)=0$, by Taylor's series method. Find $y(1.1)$.
3. State the condition for the equation

$$
A u_{x x}+2 B u_{x y}+C u_{y y}=f\left(u_{x}, u_{y}, x, y\right) \text { to be }
$$

(a) elliptic
(b) parabolic
(c) hyperbolic
when $\mathrm{A}, \mathrm{B}, \mathrm{C}$ are funetions of x and y .
4. Find the curve of best fit of the type $y=a e^{b x}$ to the following data by the method of least squares.

| x | 1 | 5 | 7 | 9 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| y | 10 | 15 | 13 | 15 | 21 |

5. Jacobi iteration scheme is used to solve the system of equations
$2 \mathrm{x}-\mathrm{y}=1$
$-\mathrm{x}+2 \mathrm{y}-\mathrm{z}=0$
$-\mathrm{y}+2 \mathrm{z}-\mathrm{w}=0$
$-\mathrm{z}+2 \mathrm{w}=1$
Find the rate of convergence of the method while starting with $x^{(0)}=[0.5,0.5,0.5,0.5]^{T}$ and iterating three times.
6. The boundary value problem $y^{\prime \prime}-2 y(x) / x^{2}=-5 / x ; 1<x<2 ; y(1)=1 ; y(2)=2$ with the h value of 0.5 .
7. (a) Using Simpson's rule find $\int 4 \mathrm{ex}+2 \mathrm{e}-\mathrm{x} d \mathrm{x}$ for given $\mathrm{e}^{0}=1, \mathrm{e}^{1}=2.72, \mathrm{e}^{2}=$ $7.39, \mathrm{e}^{3}=20.09$.
(b) State the Taylor's series formula to find $y\left(x_{1}\right)$ for solving $d y / d x=f(x, y), y_{0}$ $=\mathrm{f}\left(\mathrm{x}_{0}\right)$ and explain the assumptions used.
$[8+7]$
8. Find a root for the non-linear equation $f(x)=2 x-\log _{10} x-7$ in the interval $(2,5)$ by using regular false position method.


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1. Calculate the first and second order differences for $f(x)=a b^{c x}$
2. (a) Solve $y^{\prime}=\sin x+\cos y$ for $x=3(0.5) 4$ with the initial value of $y(0)=2.5$ using Range Kutta fourth order method.
(b) Explain the Predictor Corrector method using suitable example.
3. (a) The population of a certain town is shown in the following table,

| Year x | 1931 | 1941 | 1951 | 1961 | 1971 | 1981 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Population y | 40.62 | 60.80 | 79.95 | 103.56 | 132.65 | 142.35 |

Find the rate of growth of the population in 1961.
(b) Compare and contrast among Trepezoidal, Simpson's $1 / 3^{r d}$ rule and Simpson's $3 / 8^{\text {th }}$ rule.
4. For the following data fit a polynomial

| x | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| y | 2 | 5 | 16 | 41 |

by using
(a) Newton's backward difference formula
(b) using Legrange's interpolation formula.

Compare (a) and (b) and comment.
5. Obtain a relation of the form $y=a b^{x}$ for the following data by the method of least squares.

| x | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| y | 8.3 | 15.4 | 33.1 | 65.2 | 127.4 |

6. Write down the implicit formula to solve one dimensional heat flow equation and suggest the suitable method to solve the equations.
7. Explain the following
(a) When ill conditioning in the system is expected? Explain with an example.
(b) If $A=\left[a_{i j}\right]$ and $s_{i}=\left(a_{i 1}^{2}+a_{i 2}^{2}+\ldots \ldots+a_{i n}^{2}\right)^{1 / 2}$ then the quantity $k=\frac{|A|}{s_{1} s_{2} \ldots\left[7+s_{n}\right.}[7+8]$
indicates ill conditioning of matrix $A$.
8. (a) State the pitfalls of regular false position method.
(b) Find a real root of Wall's equation $f(x)=x^{3}-2 x-5=0$
