Code No: R09221801 **R09** **Set No. 2**

II B.Tech II Semester Examinations,APRIL 2011
MATHEMATICS-III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Derive polar form of C-R equations.
   (b) Find the regular function whose imaginary part is $\frac{x-y}{x^2+y^2}$ [15]

2. (a) Does there exist a 4 regular graph on 6 vertices? If so construct one graph.
   (b) Determine whether the following graph (Figure 1) has Hamiltonian circuit. If it does, find such circuit. [7+8]

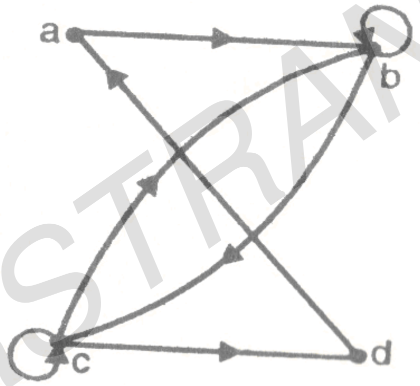

Figure 1:

3. (a) Evaluate $\int\limits_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where C: $|z| = 3/2$
   (b) Evaluate $\int\limits_c \frac{z-1}{(z+1)^2(z-2)} dz$ where C is the circle $x^2 + y^2 = 4$ [15]

4. (a) Using the method of contour integration prove that $\int\limits_0^\infty \frac{2Cos(1+x^2)}{x^{1+\alpha}} dx = 0$ $(0 < \alpha < 1)$
   (b) Find the poles and residues at those poles of the function f (z) = $\frac{z^2}{(z-1)^2(z+2)}$ [8+7]

5. (a) Find and plot the rectangular region $0 \le x \le 1, 0 \le y \le 2$ under the transformation $w = \sqrt{2}e^{i\frac{\pi}{4}}z + (1-2i)$.
   (b) Find the bilinear transformation that maps the points $\infty, i, 0$ into the points $0, i, \infty$. [7+8]

6. If $P_6(2)$= a & $P_7(2)$=b, then P.T.

(a) $P_6^1(2) = \frac{7}{3}(b-2a)$

(b) $P_8(2) = \frac{1}{8}(30b-7a)$ [15]

7. (a) P.T. $\beta\left(m+\frac{1}{2}, m+\frac{1}{2}\right) = \frac{\pi}{m.\beta(m,m)2^{4m-1}}$

   (b) S.T. $\int_0^1 \frac{x^n}{\sqrt{1-x^2}}dx = \frac{2.4.6---(n-1)}{1.3.5--n}$ where 'n' is an odd integer [15]

8. For the function f(z) = $\frac{2z^3+1}{z^2+z}$ find

   (a) Find the Taylsor's series expansion of about z =3.

   (b) Explain f(z)= Cos z in Taylor's series about z = πi. [15]

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Code No: R09221801 **R09** **Set No. 4**

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1. (a) P.T. $\int_{-1}^{1} P_n(x)dx = 0;\ n \neq 0$
   (b) P.T. $\int_{-1}^{1} P_n(x)(1 - 2xt + t^2)^{-1/2}dx = \frac{2t^n}{2n+1}$ where n is a + ve integer. [15]
2. Find Minimum spanning tree (MST) of the following weighted graph G (Figure 2) by Kruskal algorithm. [15]

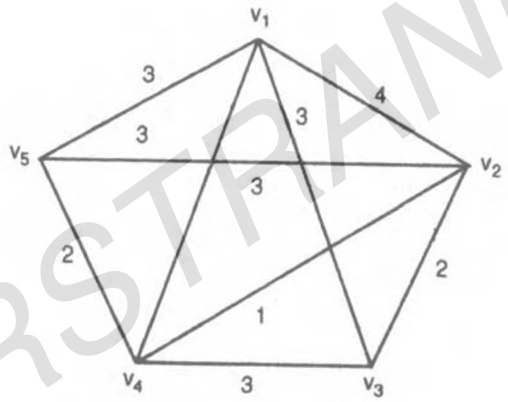

Figure 2:

3. (a) P.T. $\int_{0}^{\pi/2} \sin^2\theta \cos^4\theta d\theta = \frac{5\pi}{256}$
   (b) P.T. $\frac{d}{dx}\{J_0(x)\} = -J_1(x)$ [15]
4. (a) Find the Taylor's series expansion of f(z) = $\log\left(\frac{1+z}{1-z}\right)$ about z = 0. Also find the radius of convergence.
   (b) Expand $\frac{7z^2+9z-18}{z^3-9z}$ about $|z-3| > 6$ as Laurent's series. Find the region of convergence. [15]
5. (a) Find angle of rotation at the point $z = z + i$ when the transformation is $w = z^2$. Also find the scale factor of the transformation at that point.
   (b) In the transformation $w = i\frac{1-z}{1+z}$ show that the interior of the circle $|z| = 1$ is represented in the w-plane above the real axis. [8+7]
6. (a) Find the Residues of f (z) = $\frac{z^3}{(z-1)^4(z-2)(z-3)}$.

(b) Evaluate by the method of complex variable the integral $\int\limits_{-\infty}^{\infty} \frac{x^{\,2}}{(1+x^2)^3}\, dx$. [8+7]

7. (a) From the integral $\int\limits_{0} \frac{dz}{z+4}$ S.T $\int_0^\pi \frac{1+4\cos\theta}{17+8\cos\theta} = 0$ where C: $|z| = 1$

   (b) If C is a closed curve described in + ve sense and f(z_0)= $\int\limits_{c} \frac{z^4+z}{(z-z_0)^4} dz$ show that f(z_0) = $8\pi i z_0$ is where z_0 is a point inside 'C' and f(z_0) = 0 if z_0 lies outside 'C'. [15]

8. (a) If u is a harmonic function, then S.T. w = u^2 is not a harmonic function, unless 'u' is a constant.

   (b) Determine the analytic function whose imaginary part is
   u-v = $\frac{\cos x+\sin x-e^{-y}}{2\cos x-2\cosh y}$ & $f(\pi/2) = 0$ [15]

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1. (a) Express $P_5(x)$ as a polynomial.
   (b) P.T. $\int_0^1 P_n(x)dx = 0$ if n is even. [15]

2. (a) Evaluate using Cauchy integral function $\oint_c \frac{Cos\,\pi z}{z^2-1}\,dz$ around the rectangle 2±i, -2±i
   (b) Find the Residues of f (z) = $\frac{1+e^z}{Sinz+z\,Cosz}$ at z = 0 [8+7]

3. (a) Is there a simple graph corresponding to the following degree sequence
      i. (1, 1, 2, 3)
      ii. (2, 2, 4, 6)
   (b) Find the shortest path between the vertex 'a' and vertex 'h' in the following weighted graph (figure 3). [8+7]

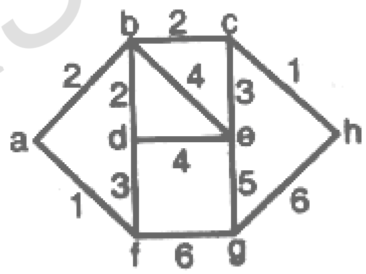

Figure 3:

4. (a) Show that the transformation $w = \frac{iz+2}{4z+i}$ transforms the real axis in the z-plane into a circle in the w-plane. Find the centre and radius of the circle.
   (b) Show that the transformation $w(z+i)^2 = 1$ transforms the inside of the circle $|z| = 1$ on the exterior of the parabola. [8+7]

5. State and prove Laurent's Theorem of complex function f (z). [15]

6. (a) Evaluate $\int_c \frac{3z^2+7z+1}{z+1}dz$ where C: $|z+i| = 1$
   (b) Evaluate $\int_c \frac{z^2-z+1}{z-1}dz$ where C: $|z| = 1/2$ taken in anticlockwise sense [15]

7. (a) S.T. J_4 (x) = $\left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x)$

(b) S.T. $\int_0^\alpha \frac{x^2}{(1+x^4)^3}dx = \frac{5\pi\sqrt{2}}{128}$ [15]

8. S.T. the function f(x,y) $= x^3y - xy^3 + xy + x + y$ can be the imaginary part of an analytic function of z=x+iy [15]

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Code No: R09221801 **R09** **Set No. 3**

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Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
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1. (a) Find the Residues of f(z) = $z^2 e^{\frac{1}{z}}$ at the pole z = 0.
   (b) Evaluate $\int\limits_c \frac{3\sin z\, dz}{\left(z^2-\frac{\pi^2}{4}\right)}$ where c is $|z| = \pi$. [8+7]

2. (a) Find the image of the circle $|z-1| = 1$ in the complex plane under the mapping w = $\frac{1}{z}$
   (b) Find the image and draw a rough sketch of the mapping of the region $1 < x < 2$ *and* $2 < y < 3$ under the mapping $w = e^z$. [7+8]

3. (a) P.T. $\int_0^\alpha e^{-y^{1/m}} dy = m\Gamma(m)$
   (b) Express J_2(x) interms of J_0(x) & J_1(x). [15]

4. (a) Evaluate $\int\limits_C \frac{e^z}{z(z+1)} dz$ where C: $|z-1| = 3$
   (b) Evaluate $\int\limits_C \frac{dz}{z(z+i\pi)}$ where C: $|z+3i| = 1$ [15]

5. (a) Prove that, for all x, $x^7 = \frac{16}{429}P_7(x) + \frac{8}{39}P_5(x) + \frac{14}{33}P_3(x) + \frac{1}{3}P_1(x)$
   (b) Show that $\int_{-1}^1 x^k P_n(x)dx = 0$ for k=0,1,2,n-1 [15]

6. (a) Represent the function f(z) = $\frac{z}{(z-3)(z-1)}$ by a series of positive and negative. Powers of (z-1), which converges to f (z) when $0 < |z-1| < 2$
   (b) Expand Sinh z by Taylor's series about z = πi
   (c) With in what circle with centre at the origin, does the Maclaurin's series for the function Tanh z converges to the function? [15]

7. Find the minimal spanning tree for the following Graph (Figure 7) using Prim's algorithm. [15]

8. (a) Find the analytic function f(z)=u+iv, given that 2u+v = $e^{2x}\{(2x+y)\cos 2y + (x-2y)\sin 2y\}$.
   (b) If (x+iy)$^{1/3}$ = a+ib, then P.T. $4(a^2-b^2) = \frac{x}{a} + \frac{y}{b}$. [15]

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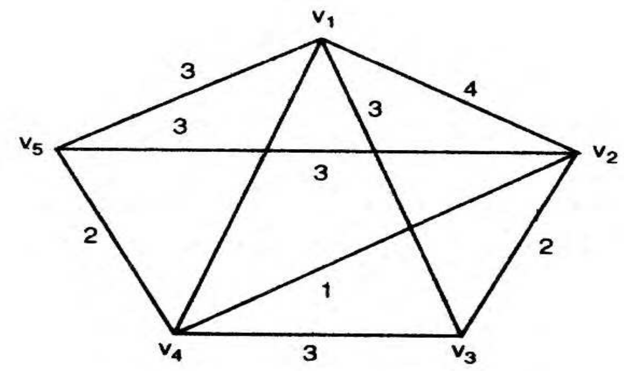

Figure 7: